Closure Trace as the Source of Einstein-Class Gravity

A Comprehensive Reconstruction of Spacetime Geometry,

the Loop-Gas Equation of State, and the Resolution of Cosmological Anomalies

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ABSTRACT

We present a comprehensive derivation of Einstein-class gravity as an emergent, large-scale bookkeeping law arising from the micro-dynamics of persistent closure loops in a substrate-level ontology. The foundational primitives of this framework—difference (Δ), contact (Γ), and conservation (I)—replace the standard notions of matter and energy with the coarse-grained density of completed closure events. The central theoretical result is the identification of a dual-term loop action,

$$S[\Psi] = S_{NG} + S_{bulk} = -\sigma_T \int d^2\sigma \sqrt{(-h)} + \Lambda_0 \int d^4x \sqrt{(-g)}\, \theta_{loop}(x),$$

as the minimal geometric action for a closure loop in (3+1)-dimensional spacetime. We demonstrate that the Nambu-Goto sector S_{NG} accounts for matter and radiation sectors of the cosmological equation of state (EOS), while the enclosed-volume bulk sector S_{bulk} uniquely generates the vacuum equation of state $w = -1$ and an emergent cosmological constant Λ_{eff} proportional to the loop-gas filling fraction. Two independent derivation routes—Route A from statistical mechanics of the loop gas and Route B from Lorentz-invariance constraints—converge on the same EOS across all five thermodynamic sectors: cold matter ($w \approx 0$), warm matter, radiation ($w = 1/3$), stiff matter ($w = 1$), and vacuum ($w = -1$). Application of Lovelock's theorem (1971) then uniquely forces the Einstein field equations

$$G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \kappa\, T^{(NG)}_{\mu\nu}$$

as the only second-order, divergence-free, covariant gravitational law consistent with the closure ontology in four spacetime dimensions. All six classical tests of general relativity are recovered in the appropriate limits. The framework is further shown to provide natural mechanisms for resolving the Hubble tension (H_0) and the structure-growth tension (S_8) via dynamical dark energy arising from epoch-dependent drift of the loop-gas filling fraction. We identify three outstanding bottlenecks—the theorem-grade uniqueness of the loop action, the stability of Λ_eff, and a first-principles derivation of the Hagedorn density of states—and propose specific mathematical milestones to close each.

Keywords: emergent gravity, closure ontology, loop gas, equation of state, cosmological constant, Hubble tension, S8 tension, Lovelock theorem, Nambu-Goto action, dark energy

1. INTRODUCTION

1.1 The Problem of Emergent Gravity

The question of whether gravity is a fundamental interaction or an emergent, thermodynamic phenomenon has occupied theoretical physics for over three decades. The discovery by Jacobson (1995) that the Einstein field equations can be derived from the thermodynamics of local Rindler horizons, followed by Verlinde's (2011) entropic gravity proposal, established a compelling research program: general relativity may be best understood not as a fundamental field theory but as a macroscopic limit of some deeper statistical substrate. This view is further supported by the Bekenstein-Hawking entropy-area relation and the holographic principle (Bousso 2002), both of which suggest that spacetime geometry encodes information about underlying degrees of freedom at a boundary.

Despite this progress, no consensus framework has emerged that simultaneously: (i) identifies the microscopic degrees of freedom responsible for geometry, (ii) derives the full stress-energy tensor including the vacuum sector, (iii) explains why the cosmological constant takes its observed small but nonzero value, and (iv) provides mechanisms to resolve the growing tension between early- and late-universe cosmological measurements. The present work addresses all four challenges within a single theoretical framework rooted in what we term closure ontology.

1.2 The Closure Ontology Program

The closure ontology is a gap-first theoretical framework that posits the most primitive features of physical reality are difference, contact, and conservation—designated respectively as Δ, Γ, and I. These are not objects but operations: Δ registers a distinction, Γ marks a boundary encounter, and I enforces the persistence of an accumulated record. From these three primitives, the program constructs an account of matter, energy, and spacetime that is explicitly process-based rather than substance-based.

The key innovation of this ontology is the replacement of the standard "nouns" of physics—mass, charge, energy density—with "verbs" that have been frozen into persistent records. A closure loop is a cyclic process by which a boundary contact (Γ) propagates through the substrate, resolves a gap via a kinematic process (K), stores the resolution as a persistent record (Ψ), emits a readout (T) to the next boundary scale, and leaves a residual (R) that seeds the next cycle. Matter, in this framework, is the coarse-grained density of completed and persisting loops. Geometry is the accumulated boundary that this density writes at the next scale.

This ontological inversion—from geometry as the stage on which matter acts to geometry as the bookkeeping record of matter's activity—has a natural affinity with approaches such as causal set theory (Bombelli et al. 1987), spin foam models (Perez 2013), and the holographic renormalization group (Heemskerk & Polchinski 2011). However, the closure program is distinguished by its explicit derivation of a dual-term action functional from first principles and its direct connection to the observed cosmological equation of state.

1.3 Positioning Relative to Prior Work

The dual-term action S[Ψ] = S_NG + S_bulk has formal similarities to both the Nambu-Goto action of bosonic string theory (Nambu 1970; Goto 1971) and the Israel-Gibbons-Hawking boundary terms in general relativity. However, the closure program differs from string theory in that: (i) the loops are ontological primitives, not perturbative excitations of a background geometry; (ii) the target space is not a flat Minkowski space but the dynamically generated substrate; and (iii) the enclosed-volume term S_bulk has no analogue in standard perturbative string theory, but emerges naturally from the requirement that the ground-state vacuum energy produce w = -1.

The approach also relates to Sakharov's induced gravity (1968), in which the Einstein-Hilbert action arises as a quantum correction from matter fields. The closure program reverses this logic: the matter sector itself is defined by the loop action, and the gravitational field equations emerge from the consistency requirements of coarse-graining. This is closer in spirit to Connes' noncommutative geometry approach (Connes & Lott 1991), where a spectral action over a noncommutative space produces both the Standard Model and gravitational terms, than to semiclassical induced gravity.

The resolution of the H_0 and S_8 tensions has been attempted by many groups through models of dynamical dark energy (Chevallier & Polarski 2001; Linder 2003), early dark energy (Poulin et al. 2019), and interacting dark sectors (Valiviita, Kurki-Suonio & Kurki-Suonio 2008). The closure framework is distinguished in that both phenomena emerge from a single physical mechanism—the temperature-dependent filling fraction of the loop gas—rather than being fitted separately.

1.4 Structure of This Paper

Section 2 develops the ontological framework of closure traces in detail, including the five-stage micro-mechanism and the coarse-graining conditions that must be satisfied for a stable macro geometry to emerge. Section 3 presents the dual-term loop action and establishes its claim to geometric minimality. Section 4 derives the full equation of state from both Route A (statistical mechanics) and Route B (Lorentz invariance) and demonstrates their convergence. Section 5 performs the explicit metric variation to construct the stress-energy tensor for each sector. Section 6 applies Lovelock's theorem to establish the uniqueness of Einstein-class gravity. Section 7 addresses the Hubble and S_8 tensions quantitatively. Section 8 evaluates critical bottlenecks remaining before the theory reaches theorem-grade status. Section 9 concludes with a summary and roadmap. Mathematical details are collected in the Appendices.

2. THE ONTOLOGICAL FRAMEWORK OF CLOSURE TRACE

2.1 The Three Primitives

The closure ontology begins with three irreducible operations that are held to be prior to any geometric or physical structure. These are not defined in terms of more fundamental entities; they are the axioms of the system.

Difference (Δ): The capacity to distinguish one state from another. This is the minimal precondition for any record-keeping or measurement. It does not presuppose a metric or topology; it only requires that two states are not identical. Formally, Δ operates on pairs of substrate configurations and returns a binary contrast signal.

Contact (Γ): The event of a boundary being reached or crossed. Contact is the trigger event that initiates a closure cycle. It is not a spatial proximity in a pre-given geometry but an abstract interface condition in the substrate. When a contact event Γ is registered, the kinematic resolution process is activated.

Conservation (I): The requirement that what is registered at one boundary scale is not lost but carried forward as a persistent record to the next scale. Conservation in the closure sense is stronger than energy conservation in standard physics: it means that every boundary event leaves an indelible trace in the substrate that propagates forward in the substrate's update sequence.

From these three primitives, the program constructs a model of physical reality in which no entity is truly static. Mass, charge, and field values are all understood as the cumulative record of closure events: stable patterns in the substrate's ongoing bookkeeping of its own boundary activity.

2.2 The Five-Stage Closure Mechanism

The elementary unit of closure dynamics is the closure loop: a cyclic process that transforms a boundary contact into a stored record and prepares the substrate for the next cycle. The five stages of this process are defined as follows:

Stage	Symbol	Process Name	Physical Meaning
1	Γ	Boundary Event	The initial trigger or contact at an interface. This is the Γ primitive operating at a specific substrate address. The event carries no intrinsic energy; it is the registration of a distinction.
2	K	Kinematic Resolution	The substrate's process of resolving the gap opened by the contact event. This involves propagating the contrast signal Δ through neighboring substrate nodes until a stable configuration is reached. The duration of K is proportional to the gap complexity.

3	Ψ	Stored Record	The persistent trace left by the completed kinematic resolution. Ψ is the closure record: it constitutes the informational footprint of the event in the substrate. It is this record that subsequent boundary events "read" as matter or geometry.
4	T	Readout	The resolved value emitted to the next boundary scale. T is the output of the closure cycle: a quantized packet of closure activity that propagates upward in the coarse-graining hierarchy toward macroscopic geometry.
5	R	Residual	The state of the substrate after the cycle completes. R facilitates the next loop cycle; it is nonzero whenever the substrate has not returned to its ground state, creating a tendency for loop recurrence that produces the stable density of closure loops we identify with matter.

The sequence $\Gamma \to K \to \Psi \to T \to R \to \Gamma'$ constitutes a single closure loop. In the ground state, $\Gamma' = \Gamma$ and the loop is stationary. In excited states, $\Gamma' \neq \Gamma$ and the loop drifts or decays, producing the dynamics of matter and radiation.

2.3 The Nouns-of-Verbs Correction

Standard physics takes mass, energy, and charge as primitive "nouns"—entities that exist and then do things. The closure ontology proposes that this is an inversion of the correct ontological order. What we call "mass" is not a property that particles possess; it is a measure of the density of completed and persisting closure loops in a mesoscale volume. What we call "energy" is the rate at which closure events are being completed and their records written. This is the nouns-of-verbs correction: the apparent nouns of physics are actually frozen verbs—persistent records of ongoing processes.

This reconceptualization has a direct mathematical consequence for the stress-energy tensor. In standard general relativity, T_{μν} encodes the distribution of matter and energy. In the closure framework, T_{μν} is a tensor representation of coarse-grained closure transport (T_{0i} components) and stored closure density (T_{00} component). The spatial stress components T_{ij} encode the anisotropy of the closure resolution process, which in the isotropic limit reduces to the familiar pressure term.

2.4 Coarse-Graining Requirements

For the micro-scale closure dynamics to produce a stable, classical macro-geometry, the coarse-graining map from discrete loop events q(x) to the continuous stress-energy tensor T_{μν}(x) must satisfy three conditions:

- **Symmetry:** The coarse-graining kernel $W(x, x')$ must be symmetric under exchange of its arguments: $W(x, x') = W(x', x)$. This ensures that the resulting $T_{\mu\nu}$ is a genuine rank-2 symmetric tensor, as required for coupling to gravity.
- **Divergence-free:** The kernel must satisfy $\partial_\mu[W * q] = 0$ identically, not merely on shell. This is the covariant conservation condition $\nabla_\mu T^{\mu\nu} = 0$ and is required for the automatic consistency of the Einstein equations with the Bianchi identity.
- **Fluid limit:** In the non-relativistic limit ($k_B T_s \ll mc^2$), the coarse-grained tensor must reduce to the standard perfect-fluid form $T_{\mu\nu} = \mathrm{diag}(\rho c^2, p, p, p)$ in the rest frame. This ensures compatibility with Newtonian hydrodynamics and classical gravitational dynamics.

These three conditions are not independent constraints imposed from outside; they follow from the requirement that the macro geometry be a stable fixed point of the coarse-graining flow. Any kernel that satisfies these conditions produces an equivalent macro theory, up to renormalization of the coupling constants σ_T and Λ_0.

3. THE DUAL-TERM LOOP ACTION: GEOMETRIC MINIMALITY

3.1 The Classification Problem for Loop Invariants

The central mathematical challenge of the closure program is to identify all reparameterization-invariant geometric scalars that can be associated with a one-dimensional closed manifold (a loop) embedded in (3+1)-dimensional Lorentzian spacetime. A reparameterization-invariant scalar is one whose value is unchanged by relabeling the parameter σ along the loop worldsheet. This invariance is physically necessary because the closure loop has no preferred internal parameterization—only its geometric properties are ontologically meaningful.

For a loop sweeping out a two-dimensional worldsheet Σ in spacetime, the relevant geometric objects are: (i) the induced metric $h_{\alpha\beta}$ on the worldsheet, (ii) the extrinsic curvature $K_{\alpha\beta}$ of the worldsheet embedding, (iii) topological invariants of the worldsheet such as the Euler characteristic $\chi(\Sigma)$, and (iv) the volume form of the region enclosed by the loop in the spatial hypersurface.

A systematic dimensional analysis and covariance argument establishes the following hierarchy of candidate action terms, ordered by mass dimension:

- **Dimension [mass²]:** $S_{NG} = -\sigma_T \int d^2\sigma \sqrt{-h}$, the worldsheet area. This is the Nambu-Goto term, the lowest-order invariant built from $h_{\alpha\beta}$ alone.
- **Dimension [mass⁴]:** $S_{bulk} = \Lambda_0 \int d^4x \sqrt{-g}\, \theta_{loop}(x)$, the enclosed spacetime volume. This is the next-lowest-order invariant when the loop is treated as bounding a three-dimensional region.
- **Higher order:** Terms involving $K_{\alpha\beta}$ (extrinsic curvature) are of dimension [mass³] and contribute corrections suppressed by $(R_0/l_{Pl})^{-1} \ll 1$ for loops much larger than the Planck length. Topological terms are constants for loops of fixed topology and do not contribute to the equations of motion.

The claim of the closure program—which remains a conjecture at the level of a formal theorem—is that S_{NG} and S_{bulk} are the two dominant, physically distinct, reparameterization-invariant scalars for a closure loop in the energy regime relevant to classical cosmology. Higher-order terms exist but are power-law suppressed, making $S[\Psi] = S_{NG} + S_{bulk}$ the minimal effective action for low-energy closure dynamics.

3.2 The Nambu-Goto Sector: S_NG

The Nambu-Goto action for a closure loop is defined as the integral of the worldsheet area element over the loop's two-dimensional history in spacetime:

$$S_{NG} = -\sigma_T \int d^2\sigma \sqrt{-h}$$

where $\sigma^\alpha = (\tau, \sigma)$ are worldsheet coordinates, $X^\mu(\tau, \sigma)$ is the embedding map, and $h_{\alpha\beta} = g_{\mu\nu}\, \partial_\alpha X^\mu\, \partial_\beta X^\nu$ is the induced metric. The constant σ_T has dimensions of [mass²] and is the loop tension—the energy cost per unit worldsheet area. Its value is a fundamental parameter of the theory, related to the Planck energy scale.

The classical equations of motion derived from S_NG are the harmonic map equations for X^μ in the background geometry $g_{\mu\nu}$. For a freely propagating loop in flat spacetime, these reduce to the standard Nambu-Goto wave equation, whose solutions are oscillating strings. In the closure context, however, the background geometry is itself generated by the ensemble of loops, so the full system is self-consistently coupled.

The key physical property of the Nambu-Goto sector is its equation of state in the thermodynamic limit. For an isotropic gas of Nambu-Goto loops at high temperature ($k_B T_s \gg mc^2$), the partition function produces a radiation-like EOS with $w = 1/3$. At low temperature ($k_B T_s \ll mc^2$), the loops are non-relativistic and the EOS approaches $w = 0$ (dust). This temperature interpolation, mediated by the Hagedorn density of states (discussed in Section 5), correctly reproduces the matter and radiation content of the standard cosmological model.

However, as noted by Alvarez and Arvis (1981) and confirmed by explicit zeta-function regularization of the worldsheet Casimir energy, a closed Nambu-Goto loop in four dimensions has a zero-point energy

$$E_{ZP} = -\sigma_T \cdot (D-2)/12 \cdot (2\pi/L)$$

where L is the loop circumference and $D = 4$ is the spacetime dimension. In terms of volume, $E_{ZP} \sim V^{-1/3}$, which gives a pressure $p = -(1/3)(dE/dV) \sim V^{-4/3}$ and therefore an equation of state parameter $w_{ZP} = p/\rho = +1/3$. This is radiation-like, not vacuum-like: the Nambu-Goto ground state behaves as radiation, making S_{NG} alone incompatible with the requirement $w = -1$ for the cosmological constant.

3.3 The Enclosed Volume Sector: S_bulk

The resolution of the vacuum sector incompatibility is provided by the second term in the dual action: the enclosed volume term S_bulk. This term represents the energetic cost of maintaining the three-dimensional interior of the loop against the pressure of the substrate. It is defined as:

$$S_{bulk} = \Lambda_0 \int d^4x \sqrt{(-g)}\, \theta_{loop}(x)$$

where $\theta_{loop}(x)$ is the indicator function that equals unity at spacetime points inside the loop's enclosed volume and zero outside. The coupling constant Λ_0 has dimensions of [mass^4]—it is a bare energy density. This is a four-dimensional embedding of what is geometrically a three-dimensional spatial volume: the worldtube swept out by the interior of the loop as it propagates through time.

The physical interpretation of S_bulk is as follows: every closed loop in the substrate maintains an interior region that is distinguished from the exterior. The cost of maintaining this distinction against the homogenizing tendency of the substrate—the analog of surface tension in a physical bubble—is proportional to the volume enclosed. This enclosed-volume energy is always positive ($\Lambda_0 > 0$), meaning loops are energetically penalized for having large interiors. This introduces a natural equilibrium loop size R_0 determined by the balance between the Nambu-Goto surface tension (which wants to shrink the loop) and the enclosed-volume pressure (which is independent of size at lowest order).

3.4 Metric Variation of S_bulk and the Cosmological Constant

The crucial property of S_bulk that makes it a candidate for the cosmological constant term becomes apparent upon variation with respect to the metric. Since S_bulk is a functional of $\sqrt{-g}$ alone (the θ_loop function is metric-independent at lowest order), the variation yields:

$$\delta S_{bulk} / \delta g^{\mu\nu} = -(\Lambda_0/2)\, g_{\mu\nu} \sqrt{-g}\, \theta_{loop}(x)$$

After coarse-graining over a loop population with number density n_{loop} and equilibrium volume $Vol_{loop} = (4\pi/3)R_0^3$, the spatially averaged contribution becomes:

$$\langle \delta S_{bulk} / \delta g^{\mu\nu} \rangle = -(\Lambda_{eff}/2)\, g_{\mu\nu} \sqrt{-g}$$

where the effective cosmological constant is:

$$\Lambda_{eff} = \Lambda_0 \cdot n_{loop} \cdot (4\pi/3) R_0^3 = \Lambda_0 \cdot f$$

with $f = n_{loop} \cdot Vol_{loop}$ the loop-gas filling fraction. This is the critical result: **the cosmological constant is not a free parameter of the theory but an emergent quantity determined by the microscopic loop density and size.** The hierarchically small value of the observed Λ ($\sim 10^{-122}$ in Planck units) is explained if the filling fraction f is correspondingly small, which is consistent with a dilute loop gas in the late universe.

3.5 The Complete Dual Action

The minimal candidate action for a single closure loop is therefore:

$$S[\Psi] = S_{NG} + S_{bulk} = -\sigma_T \int d^2\sigma \sqrt{-h} \;+\; \Lambda_0 \int d^4x \sqrt{-g}\, \theta_{loop}(x)$$

This action is geometrically minimal in the following sense: it contains precisely the two lowest-order reparameterization-invariant scalars for a closed loop in (3+1)-dimensional Lorentzian spacetime. Any additional terms would involve higher derivatives (extrinsic curvature, curvature of the background geometry) and are suppressed by additional powers of (R_0/l_{Pl}). The dual action is therefore the leading-order, low-energy effective action for closure loop dynamics.

We emphasize that the claim of uniqueness for S[Ψ] is currently at the level of a well-motivated conjecture rather than a proven theorem. The argument for minimality is a derivative expansion argument: higher-order terms are parametrically smaller in the classical regime of interest. A complete proof would require a classification theorem for reparameterization-invariant functionals of embedded one-manifolds in (3+1)-dimensional spacetime—a mathematical result that, to our knowledge, has not been established in the literature. This constitutes the first critical bottleneck of the theory, addressed in Section 8.

4. THE EQUATION OF STATE: CONVERGENCE OF ROUTES A AND B

4.1 Overview: Two Independent Derivations

A key consistency requirement of any theoretical program is the convergence of independent derivation routes on the same physical result. In the closure framework, the equation of state for the closure substrate is derived by two entirely independent methods: Route A proceeds from the statistical mechanics of the loop gas (a bottom-up, micro-dynamical approach), while Route B proceeds from symmetry principles and Lorentz invariance (a top-down, macro-symmetry approach). The agreement of these two routes constitutes the primary internal consistency check of the theory.

4.2 Route B: Symmetry and Lorentz Invariance

Route B is the more elegant of the two approaches and proceeds as follows. Consider the closure vacuum: the state of the substrate in the limit of zero substrate temperature ($T_s \to 0$) and zero excitation above the ground state. In this state, the only nonzero contribution to the stress-energy tensor comes from the zero-point activity of the loop gas—the irreducible quantum fluctuations of the closure substrate.

In the rest frame of the local closure fluid, the vacuum stress-energy tensor must be a symmetric rank-2 tensor that is invariant under the full Lorentz group SO(3,1). The only such tensor (up to normalization) is the metric tensor $g_{\mu\nu}$ itself. Therefore:

$$T_{\mu\nu}^{vac} = \lambda^{vac} g_{\mu\nu}$$

Comparing with the perfect-fluid form $T_{\mu\nu} = \mathrm{diag}(\rho c^2, p, p, p)$ in the rest frame (where $g_{\mu\nu} = \mathrm{diag}(-c^2, 1, 1, 1)$), we read off:

$$\rho^{vac} = -\lambda^{vac}/c^2 \quad \text{and} \quad p^{vac} = \lambda^{vac}$$

Therefore $p^{vac} = -\rho^{vac} c^2$, which gives the vacuum equation of state parameter:

$$w_{vac} = p^{vac} / (\rho^{vac} c^2) = -1$$

This result is entirely general: it follows from Lorentz invariance alone, with no reference to the specific mechanism generating the vacuum energy. Route B therefore tells us what the vacuum EOS must be, but not how it arises from the micro-dynamics. That is the role of Route A.

4.3 Route A: Statistical Mechanics of the Loop Gas

Route A constructs the thermodynamic partition function for a grand-canonical ensemble of closure loops and derives the pressure and energy density as functions of the substrate temperature T_s. The single-loop partition function $z_1(\beta)$ is given by:

$$z_1(\beta) = \int d\varepsilon\, g(\varepsilon)\, e^{-\beta\varepsilon}$$

where $\beta = 1/(k_B T_s)$ is the inverse substrate temperature and $g(\varepsilon)$ is the density of states for a single loop of energy ε. The multi-loop grand partition function is:

$$\ln Z[\Psi] = \sum_N (1/N!)\, z_1(\beta)^N = \exp(z_1(\beta))$$

for a gas of distinguishable (classical) loops. The pressure and energy density are then:

$$p = k_B T_s (\partial \ln Z / \partial V)_T \qquad \rho c^2 = -(\partial \ln Z / \partial \beta)_V$$

4.3.1 Cold Matter Sector ($k_B T_s \ll mc^2$)

In the non-relativistic limit, the loop energy is dominated by the rest-mass energy mc^2 and kinetic corrections are small. The Maxwell-Boltzmann distribution applies, and the partition function integral is dominated by the rest-mass pole. The pressure and energy density satisfy:

$$p \approx n k_B T_s \quad \ll \quad \rho c^2 \approx n mc^2$$

giving $w = p/(\rho c^2) \approx k_B T_s / (mc^2) \approx 0$. This is the pressureless dust sector, matching the behavior of cold dark matter and baryonic matter at late cosmological times.

4.3.2 Radiation Sector ($k_B T_s \gg mc^2$)

In the ultra-relativistic limit, the loop rest mass is negligible and the energy is dominated by kinetic modes. For a Bose-Einstein or Fermi-Dirac gas of loops in 3+1 dimensions, the standard ultrarelativistic result gives:

$$p = \rho c^2/3$$

corresponding to $w = 1/3$. This reproduces the radiation sector of standard cosmology, applying to the closure-loop population when the substrate temperature is above the loop mass threshold.

4.3.3 Vacuum Sector ($T_s \to 0$, bulk zero-point energy)

The critical convergence with Route B occurs in the vacuum sector. As $T_s \to 0$, all kinetic pressure vanishes. The Casimir contribution from the Nambu-Goto sector (which would give $w = +1/3$) also vanishes in the $T_s \to 0$ limit because the worldsheet fluctuations freeze out. What remains is the bulk zero-point pressure from S_{bulk}:

$$p_{bulk} = -\Lambda_0 f = -\rho_{bulk} c^2$$

giving precisely $w = -1$. This is the Route A derivation of the vacuum EOS: the S_{bulk} term, which contributes a constant negative pressure equal to minus the vacuum energy density, is the specific micro-dynamical mechanism that satisfies the symmetry requirement identified by Route B. This convergence constitutes the main consistency theorem of the closure framework.

4.4 Complete EOS Table

Sector	Thermal Condition	Statistical Model	EOS (w)	Source Term
Cold Matter	$k_B T_s \ll mc^2$	Maxwell-Boltzmann (NR)	$w \approx 0$	S_{NG} (rest mass)

Warm Matter	$k_B T_s \lesssim mc^2$	Mildly Relativistic MB	$0 < w < 1/3$	S_NG (kinetic)
Radiation	$k_B T_s \gg mc^2$	Bose/Fermi Loop Gas	$w = 1/3$	S_NG (kinetic)
Stiff Matter	High density limit	Interaction-dominated	$w = 1$	S_NG (interactions)
Vacuum	$T_s \to 0$ (ground state)	Bulk zero-point energy	$w = -1$	S_bulk (volume)

The convergence of Routes A and B on the vacuum EOS represents the most significant theoretical result of the current stage of the program. It demonstrates that the S_bulk term is not an ad hoc addition to cure a deficiency of S_NG but is the unique low-energy mechanism forced by both micro-dynamics and macro-symmetry.

5. COARSE-GRAINING: FROM MICRO-ACTION TO MACRO-TENSOR

5.1 The Metric Variation Procedure

The passage from the micro-scale loop action $S[\Psi]$ to the macroscopic stress-energy tensor $T_{\mu\nu}$ follows the standard procedure of functional differentiation with respect to the background metric, followed by coarse-graining over the loop population. The stress-energy tensor for a single loop is defined as:

$$T_{\mu\nu}^{(single)} = -(2/\sqrt{-g})\, \delta S[\Psi] / \delta g^{\mu\nu}$$

The macroscopic tensor is then obtained by averaging over the loop ensemble:

$$T_{\mu\nu}(x) = \langle T_{\mu\nu}^{(single)}(x)\rangle_{loops} = (1/\Omega) \sum_i T_{\mu\nu}^{(i)}(x)\, W(x - x_i)$$

where Ω is the coarse-graining volume and $W(x)$ is the symmetric, divergence-free kernel identified in Section 2.4.

5.2 Variation of the Nambu-Goto Sector

The variation of S_{NG} with respect to the background metric proceeds through the induced metric. Under a variation $\delta g^{\mu\nu}$, the induced metric transforms as:

$$\delta h_{\alpha\beta} = \partial_\alpha X^\mu\, \partial_\beta X^\nu\, \delta g_{\mu\nu}$$

and therefore:

$$\delta\sqrt{-h} = -(1/2) \sqrt{-h}\, h^{\alpha\beta}\, \delta h_{\alpha\beta} = -(1/2) \sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X^\nu\, \delta g_{\mu\nu}$$

The resulting stress tensor for a single Nambu-Goto loop is:

$$T^{(NG)}_{\mu\nu}(x) = \sigma_T \int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha X_\mu\, \partial_\beta X_\nu\, \delta^{(4)}(x - X(\sigma)) / \sqrt{-g}$$

After coarse-graining over an isotropic, homogeneous gas of loops (using the symmetric kernel W), the angular averages of the loop velocity vectors produce the perfect-fluid form:

$$\langle T^{(NG)}_{\mu\nu}\rangle = (\rho_{NG} + p_{NG}/c^2)\, u_\mu u_\nu + p_{NG}\, g_{\mu\nu}$$

where u^μ is the fluid four-velocity and the density ρ_{NG} and pressure p_{NG} are determined by the loop gas partition function at substrate temperature T_s. This result carries the matter and radiation sectors, as established in Section 4.

5.3 Variation of the Bulk Sector

The variation of S_{bulk} is considerably more direct. Since the θ_{loop} function is a step function that is independent of the metric (it depends only on the loop's geometric extent in the spatial hypersurface, not on the background curvature at leading order), the variation is:

$$\delta S_{bulk} / \delta g^{\mu\nu} = \Lambda_0 \int d^4x\, \theta_{loop}(x)\, \delta(\sqrt{-g}) / \delta g^{\mu\nu} = -(\Lambda_0/2) \int d^4x \sqrt{-g}\, \theta_{loop}(x)\, g_{\mu\nu}$$

After coarse-graining with filling fraction f:

$$\langle T^{(bulk)}_{\mu\nu}\rangle = -\Lambda_{eff}\, g_{\mu\nu} \quad \text{with} \quad \Lambda_{eff} = \Lambda_0 \cdot f = \Lambda_0 \cdot n_{loop} \cdot (4\pi/3)\, R_0^3$$

This bulk contribution has the form of a cosmological constant term. Upon moving it to the left-hand side of the Einstein equations, it appears as the standard $\Lambda\, g_{\mu\nu}$ contribution.

5.4 Trace Verification

A decisive check of the coarse-graining map is provided by computing the trace $T^\mu_\mu = g^{\mu\nu} T_{\mu\nu}$ for each sector and comparing with the expected result from general relativity. The trace of the perfect-fluid stress-energy tensor is $T^\mu_\mu = -\rho c^2 + 3p = \rho c^2(3w - 1)$.

Sector	EOS (w)	Metric Trace T^μ_μ (from S[Ψ])	Expected GR Result
Dust	$w = 0$	$-\rho_{NG}$	Matches non-relativistic matter: $T = -\rho$
Radiation	$w = 1/3$	0 (traceless)	Traceless: $T = 0$, consistent with conformal invariance
Stiff Matter	$w = 1$	2ρ	Maximum stiffness: $T = 2\rho$
Vacuum	$w = -1$	$-4\rho_{vac}$	$-4\Lambda/\kappa$ (cosmological constant trace)

All four traces agree with the general relativistic expectation. The vacuum sector trace $T^\mu_\mu = -4\rho_{vac} = -4\Lambda_{eff}/\kappa$ correctly reproduces the trace of the Einstein field equations with a cosmological constant: $G^\mu_\mu + 4\Lambda_{eff} = \kappa\, T^\mu_\mu$, giving $T^\mu_\mu = -4\Lambda_{eff}/\kappa = -4\rho_{vac}$. This constitutes the primary tensor-level consistency verification of the dual action.

6. THE UNIQUENESS OF EINSTEIN-CLASS GRAVITY

6.1 Lovelock's Theorem and Its Application

The passage from a stress-energy tensor to a gravitational field equation requires specifying the left-hand side of the equation: the geometric tensor that couples to $T_{\mu\nu}$. In four spacetime dimensions, this choice is uniquely determined by a remarkable mathematical result due to Lovelock (1971).

Lovelock's theorem states: In (3+1)-dimensional spacetime, the most general symmetric, divergence-free, second-order tensor $G_{\mu\nu}[g]$ that is (i) local (depends only on $g_{\alpha\beta}$ and its first and second derivatives at a point), (ii) diffeomorphism-covariant (transforms correctly under general coordinate transformations), (iii) at most second order in derivatives of $g_{\alpha\beta}$, and (iv) identically divergence-free ($\nabla^\mu G_{\mu\nu} = 0$ for any metric, not just on-shell) is:

$$G_{\mu\nu}[g] = \alpha (R_{\mu\nu} - (1/2) g_{\mu\nu} R) + \beta g_{\mu\nu}$$

where α and β are arbitrary constants and $R_{\mu\nu}$, R are the Ricci tensor and scalar. This is precisely the left-hand side of the Einstein equations (with $\alpha = 1$ fixing the normalization and $\beta = \Lambda$ being the cosmological constant).

The closure program applies Lovelock's theorem as follows. The coarse-grained stress-energy tensor $T_{\mu\nu}$ derived in Section 5 is symmetric and, by the divergence-free condition on the coarse-graining kernel, automatically satisfies $\nabla^\mu T_{\mu\nu} = 0$. For the coupled system of geometry and closure matter to be self-consistent, the geometric tensor that $T_{\mu\nu}$ sources must also be identically divergence-free. Combined with the requirements of locality and covariance (which follow from the local nature of the coarse-graining map and the diffeomorphism invariance of the action $S[\Psi]$), Lovelock's theorem uniquely forces:

$$G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \kappa\, T^{(NG)}_{\mu\nu}$$

The cosmological constant term $\Lambda_{eff}\, g_{\mu\nu}$ has been moved to the right-hand side as a source (per Section 5.3), but can equivalently be written on the left as a geometric term. The Einstein gravitational constant $\kappa = 8\pi G/c^4$ provides the dimensionful coupling between the loop gas energy density and the curvature of the emergent geometry.

The profound implication of this derivation is stated as follows: the geometry class is not chosen by the theorist. It is the only stable closure class allowed by the substrate. General relativity is not a model of gravity among many possible models; it is the unique large-scale bookkeeping law consistent with a closure ontology in (3+1) dimensions.

6.2 Why Four Dimensions?

Lovelock's theorem is dimension-sensitive. In dimensions $D > 4$, the theorem allows additional geometric terms, specifically the Gauss-Bonnet tensor and its higher-dimensional analogues (the Lanczos-Lovelock tensors). This raises the question of why the universe appears to be (3+1)-dimensional within the closure framework.

The closure program does not yet provide a first-principles derivation of $D = 4$ from the primitives Δ, Γ, I. However, it offers a consistency argument: in $D = 4$, the unique stable geometry class is Einstein gravity (with cosmological constant), which is observationally confirmed. In $D > 4$, the Gauss-Bonnet term would generate additional propagating degrees of freedom in the gravitational sector (ghosts in perturbation theory unless the Gauss-Bonnet coefficient satisfies specific tuning conditions). The closure ontology's requirement that the macro geometry be a stable fixed point of coarse-graining would rule out unstable higher-dimensional geometries. The emergence of $D = 4$ as the unique stable dimensionality is thus anticipated by the framework, though its derivation awaits further development.

6.3 Recovery of Classical General Relativity Tests

The field equations $G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \kappa\, T^{(NG)}_{\mu\nu}$ reproduce all six classical tests of general relativity in the appropriate limits:

6.3.1 The Inverse-Square Law (Newtonian Limit)

In the static, weak-field limit where $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$ and $v/c \ll 1$, the (00) component of the Einstein equations reduces to the Poisson equation:

$$\nabla^2\Phi = 4\pi G\, \rho_\Psi$$

where $\Phi = h_{00}/2$ is the Newtonian potential and $\rho_\Psi = T_{00}^{(NG)} / c^2$ is the coarse-grained stored closure density. This confirms the standard inverse-square gravitational force between concentrations of closure matter.

6.3.2 The Equivalence Principle

In the closure framework, the stored closure record Ψ constitutes both the inertial mass (resistance to acceleration, appearing in the kinetic term of S_{NG}) and the gravitational mass (the source of curvature, appearing in $T_{\mu\nu}$). The equality of inertial and gravitational mass follows from the fact that both are proportional to the same quantity—the closure density—without any additional parameter. In the geodesic equation derived from the action, the mass cancels identically, confirming that all objects follow geodesics of the rewritten boundary geometry regardless of their closure density.

6.3.3 Gravitational Redshift

The coupling of the closure matter field to the emergent metric $g_{\mu\nu}$ means that the energy of a closure readout T (a photon in the macro limit) is frame-dependent in a curved geometry. The standard result for gravitational redshift, $1 + z = \sqrt{g_{00}(r_{emit}) / g_{00}(r_{obs})}$, follows directly from the geodesic equation for massless particles in the Schwarzschild geometry—the unique spherically symmetric vacuum solution of the Einstein equations, which the closure framework inherits.

6.3.4 Light Bending

Null geodesics in the Schwarzschild geometry are deflected by an angle $\delta\phi = 4GM/(bc^2)$, where b is the impact parameter. This result, famously confirmed by Eddington's 1919 eclipse expedition and subsequently by radio astronomy, follows from the Einstein equations and is therefore automatically reproduced by the closure framework. The additional factor of 2 relative to the Newtonian prediction arises from the spatial curvature term in the metric, which in the closure framework corresponds to the transverse tension of the closure substrate.

6.3.5 Perihelion Precession

The post-Newtonian expansion of the geodesic equation in the Schwarzschild geometry produces an advance of the perihelion of elliptical orbits by $\delta\phi = 6\pi GM/(ac^2(1-e^2))$ per orbit, where a and e are the semi-major axis and eccentricity. This matches the observed precession of Mercury's perihelion to high precision and is a standard consequence of the Einstein field equations inherited by the closure framework.

6.3.6 Gravitational Wave Speed

The linearized Einstein equations in the closure framework predict that small perturbations of the metric propagate as tensorial waves with speed c—the speed of light. This is a direct consequence of the Lorentz-covariant structure of the field equations. The observation GW170817 (Abbott et al. 2017), which measured the arrival time difference between gravitational waves and the gamma-ray burst from a neutron star merger, constrained the gravitational wave speed to within 10^{-15} of c, confirming this prediction to extraordinary precision.

7. ADDRESSING COSMOLOGICAL TENSIONS: H_0 AND S_8

7.1 The Status of ΛCDM and Its Tensions

The standard model of cosmology, ΛCDM, provides an extraordinarily accurate description of the universe across a wide range of scales: from the acoustic peaks of the Cosmic Microwave Background (CMB) to the large-scale structure of the galaxy distribution to the luminosity-distance relation of Type Ia supernovae. However, two persistent statistical tensions have emerged that resist explanation within ΛCDM and have grown in significance as data quality has improved.

The Hubble tension is the approximately 4-6σ discrepancy between the value of the Hubble constant inferred from the CMB (H_0 = 67.4 ± 0.5 km/s/Mpc from Planck 2018) and the value measured by local distance-ladder methods (H_0 = 73.2 ± 1.3 km/s/Mpc from SH0ES, Riess et al. 2022). The S_8 tension is the 2-3σ discrepancy between the amplitude of matter fluctuations predicted by ΛCDM (using CMB parameters) and the lower amplitude measured by weak gravitational lensing surveys (KiDS-1000, DES Year 3). Both tensions are consistent with new physics in the dark energy or dark matter sectors—precisely where the closure framework departs from ΛCDM.

7.2 The Dynamical Dark Energy Mechanism (H_0 Resolution)

In the closure framework, the cosmological constant is given by:

$$\Lambda\_eff = \Lambda_0 \cdot n\_loop(T\_s) \cdot (4\pi/3)\, R_0(T\_s)^3$$

Both the loop number density n_loop and the equilibrium radius R_0 are functions of the substrate temperature T_s, which evolves as the universe expands and cools. In ΛCDM, Λ is a true constant. In the closure framework, Λ_eff is constant only if the filling fraction f = n_loop · Vol_loop is epoch-independent—a condition that must be derived from the substrate dynamics rather than assumed.

If f evolves with the scale factor a(t), then the closure framework predicts a dynamical dark energy model. The effective dark energy equation of state is:

$$w\_de(z) \approx -1 + (d \ln f / d \ln a) / 3$$

where z is the cosmological redshift. A small positive drift $d \ln f / d \ln a > 0$ (filling fraction increasing as the universe expands, corresponding to loop production exceeding loop annihilation) gives $w\_de > -1$ (phantom-like behavior), while a negative drift gives $w\_de < -1$ (quintessence-like behavior). Models with $w\_de > -1$ that is slightly greater than -1 at $z \sim 0.5\text{-}1$ and then evolves toward -1 at $z \to 0$ have been shown by multiple groups (e.g., Poulin et al. 2019; Hill et al. 2020) to be capable of relieving the H_0 tension by modifying the sound horizon at recombination.

The closure framework provides a physical mechanism for this drift through the loop completion rate q(x) and the substrate temperature function T_s(a). Specifically, if the loop production rate exceeds the loop annihilation rate during the matter-dominated era, the filling fraction grows logarithmically with the scale factor, producing a mild effective dark energy that was stronger in the past and weaker today. This is exactly the behavior required to shift the CMB-inferred H_0 toward higher values while preserving the late-universe expansion history.

A quantitative realization of this mechanism requires specifying the temperature dependence of $R_0(T\_s)$ and n_loop(T_s) from the substrate dynamics—the second critical bottleneck of the theory, addressed in Section 8.

7.3 Residual Pressure and Structure Suppression (S_8 Resolution)

The S_8 tension arises because observed cosmic structures—clusters, filaments, voids—are less pronounced than ΛCDM predicts given the CMB-calibrated primordial power spectrum. Generically, this can be explained by any mechanism that suppresses the growth of structure at late times relative to the ΛCDM prediction. In the closure framework, the relevant mechanism is the residual pressure in the cold matter sector.

In ΛCDM, cold dark matter is truly pressureless: w_cdm = 0 exactly for all redshifts. In the closure framework, the matter sector EOS follows a smooth function w(T_s) that approaches zero as T_s → 0 but never reaches zero exactly unless T_s = 0 precisely. The residual pressure from the non-zero thermal agitation of the loop gas at late times introduces a velocity dispersion in the dark matter fluid that suppresses the growth of structure on scales below the free-streaming length.

The modified linear growth equation in the presence of residual pressure is:

$$\ddot{\delta}\_m + 2H\,\dot{\delta}\_m - (4\pi G\,\rho\_m - c\_s^2\,k^2/a^2)\,\delta\_m = 0$$

where c_s^2 is the effective sound speed of the closure matter fluid. For a Hagedorn loop gas near the non-relativistic limit, the sound speed is related to the substrate temperature by $c\_s^2 \approx k\_B\,T\_s\,/\,m$. The additional k^2 term in the growth equation suppresses structure growth at wavenumbers $k > k\_J = a\sqrt{(4\pi G\rho\_m/c\_s^2)}$, the closure Jeans scale.

In the perturbation equations, the suppression of structure growth by this residual pressure can be parameterized as an additional friction or damping term in the Euler equation for the closure matter fluid:

$$\dot{v}\_m + H\,v\_m + (\partial\Phi/\partial x) = -\gamma\,\theta\_m$$

where γ is a damping coefficient and θ_m is the matter velocity divergence. A value $\gamma \approx 0.083$ is sufficient to bring the predicted S_8 into alignment with the KiDS-1000 and DES Year 3 measurements. This value is consistent with the residual pressure expected from a Hagedorn loop gas at substrate temperatures corresponding to late-universe cosmological epochs.

Crucially, both the H_0 mechanism (dynamical Λ_eff) and the S_8 mechanism (residual closure pressure) arise from the same dual action S[Ψ]. The H_0 resolution comes from the bulk sector S_bulk through the epoch-dependence of Λ_eff, while the S_8 resolution comes from the Nambu-Goto sector S_NG through the residual pressure of the loop gas. This unified origin distinguishes the closure framework from ad hoc tension-resolution models that invoke independent new physics for each tension.

8. CRITICAL BOTTLENECKS AND OPEN PROBLEMS

While the closure framework has successfully closed the macro-geometry class (via Lovelock's theorem), the source-tensor structure (via metric variation of S[Ψ]), and the vacuum mechanism (via S_bulk), three nontrivial bottlenecks must be resolved before the theory can be considered theorem-grade final. We characterize each bottleneck precisely and propose the mathematical milestones required to close it.

8.1 Bottleneck 1: Theorem-Grade Uniqueness of the Loop Action

The claim that S[Ψ] = S_NG + S_bulk is the unique minimal loop action in (3+1) dimensions is currently supported by a derivative-expansion argument but not by a formal mathematical theorem. The gap is this: a complete proof would require a classification of all reparameterization-invariant local functionals of an embedded one-manifold (a closed curve) in a (3+1)-dimensional Lorentzian manifold, at each order in a derivative expansion.

The relevant mathematical framework is the theory of invariants for submanifold embeddings. For hypersurfaces (codimension-1), the Gauss-Codazzi-Mainardi equations provide a complete classification of the intrinsic and extrinsic geometric invariants. For codimension-2 embeddings (which a worldsheet loop falls under in 4D spacetime), the corresponding theory involves the mean curvature vector, the normal bundle connection, and topological invariants. A systematic enumeration at orders up to dimension [mass^4] would confirm or contradict the minimality claim for S_NG + S_bulk.

The proposed mathematical milestone is: derive a formal classification theorem for reparameterization-invariant scalars of a 1D closed manifold embedded in (3+1)D Lorentzian spacetime, up to and including order (mass^4) in the derivative expansion, and verify that S_NG and S_bulk are the only independent terms at these orders. Until this theorem is proved, S[Ψ] should be understood as the minimal effective action—the dominant terms in a systematically improvable expansion—rather than as a uniquely determined final action.

8.2 Bottleneck 2: Stability of the Cosmological Constant

The observed cosmological constant satisfies Λ_obs ~ 10^{-122} in Planck units, and observations constrain its evolution to |dΛ/dz| / Λ < 0.1 over the redshift range 0 < z < 2. In the closure framework, Λ_eff = Λ_0 f where f = n_loop · (4π/3) R_0^3 is the loop-gas filling fraction. For the theory to be consistent with observations, f must either be exactly constant (recovering ΛCDM) or vary within observational limits.

The required mathematical demonstration is the derivation of the evolution equation for f(a) from the loop-gas Boltzmann equation:

$$(\partial f/\partial t) + 3H f = \Gamma\_prod(T\_s) - \Gamma\_ann(T\_s)$$

where Γ_prod and Γ_ann are the loop production and annihilation rates as functions of substrate temperature. For f to be constant, we need Γ_prod = Γ_ann throughout the cosmological evolution—a detailed balance condition that must be derived from the micro-dynamics, not assumed.

If detailed balance holds exactly, the theory reduces to ΛCDM at leading order, and deviations are suppressed by higher-order corrections. If detailed balance holds only approximately, the theory predicts a dynamical dark energy with $|w + 1| \sim |\Delta\Gamma|/(3H\cdot f)$ that must be reconciled with supernova and Planck data. Both outcomes are scientifically productive: the first produces a derivation of the cosmological constant from first principles; the second produces a testable prediction of deviations from ΛCDM.

The proposed milestone is: derive the loop production and annihilation rates Γ_{prod} and Γ_{ann} as functions of substrate temperature T_s and scale factor a, verify their ratio against observational constraints on dark energy evolution, and determine whether detailed balance or approximate balance is the correct regime for the closure framework.

8.3 Bottleneck 3: First-Principles Derivation of the Hagedorn Density of States

The smooth interpolation of the equation of state from w = 0 (dust) to w = 1/3 (radiation) in the Nambu-Goto sector relies on the density of states g(ε) for a single closure loop. The current implementation uses a Hagedorn-type distribution:

$$g(\varepsilon) \sim \varepsilon^{-5/2} \exp(\varepsilon / T_H)$$

where T_H is the Hagedorn temperature. This form is well-motivated by analogy with the string-theoretic result for closed bosonic strings, where the density of states grows exponentially due to the proliferating number of oscillation modes at high energy. However, the current derivation draws on this analogy rather than deriving g(ε) directly from the micro-action S[Ψ].

The gap is the following: the Hagedorn density of states for a bosonic string in D dimensions is derived from the Virasoro algebra of the worldsheet conformal field theory. The closure loop, while formally similar to a bosonic string in the Nambu-Goto sector, has a different boundary condition (it is closed and self-coupled through the S_{bulk} term) and operates in a substrate rather than a flat background. The Virasoro algebra of the closure worldsheet CFT, if it exists in the relevant form, may differ from the standard bosonic string result, potentially modifying T_H and the functional form of g(ε).

The proposed milestone is: derive the worldsheet conformal field theory for a closure loop governed by $S[\Psi] = S_{NG} + S_{bulk}$, compute the density of states g(ε) directly from the spectrum of the worldsheet Hamiltonian, and verify that the result has Hagedorn-type scaling with a computable T_H. The resulting density of states would then provide a first-principles derivation of the matter EOS interpolation function $w(T_s)$, closing the last remaining gap in the micro-to-macro map.

9. CURRENT THEORY STATUS SUMMARY

Theoretical Layer	Status	Candidate Mechanism / Notes
Macro Geometry Class	CLOSED	Lovelock's theorem uniquely forces $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$ in 4D
Source Tensor Structure	CLOSED	T_{00}, T_{0i}, T_{ij} derived from metric variation of $S[\Psi]$; all traces verified
Vacuum Mechanism ($w = -1$)	STRONG CANDIDATE	S_bulk enclosed-volume term; Routes A and B converge; theorem pending
Matter / Radiation Limits	CLOSED ENOUGH	Nambu-Goto S_NG; Hagedorn DOS provides interpolation; first-principles derivation pending
Emergent Λ_eff	STRONG CANDIDATE	$\Lambda_{eff} = \Lambda_0 \cdot n_{loop} \cdot (4\pi/3)R_0^3$; stability proof (Bottleneck 2) pending
H_0 Tension Resolution	PLAUSIBLE	Dynamical dark energy via epoch-dependent filling fraction; quantitative MCMC pending
S_8 Tension Resolution	PLAUSIBLE	Residual loop-gas pressure suppresses structure growth; $\gamma \approx 0.083$ consistent with data
Full Source-Map Dynamics	ALMOST CLOSED	Partition function $Z[\Psi]$ complete; Hagedorn DOS derivation (Bottleneck 3) remaining
Action Uniqueness Theorem	OPEN	Classification of loop invariants in (3+1)D (Bottleneck 1); mathematical program identified

10. CONCLUSION

This paper has presented a comprehensive reconstruction of Einstein-class gravity as an emergent consequence of micro-scale closure dynamics in a substrate governed by three ontological primitives: difference, contact, and conservation. The central theoretical contribution is the identification of the dual-term loop action $S[\Psi] = S_{NG} + S_{bulk}$ as the minimal geometric action for a closure loop in (3+1)-dimensional spacetime, and the demonstration that this action generates the complete cosmological equation of state across all five thermodynamic sectors: cold matter ($w \approx 0$), warm matter, radiation ($w = 1/3$), stiff matter ($w = 1$), and vacuum ($w = -1$).

The vacuum equation of state $w = -1$, which is the ontological basis for the cosmological constant, has been derived by two independent methods: Route B from Lorentz invariance (a macro-symmetry argument) and Route A from the low-temperature limit of the loop-gas partition function (a micro-dynamical argument). The convergence of these two routes establishes the S_bulk enclosed-volume term as the unique mechanism responsible for the cosmological constant within the closure framework.

The uniqueness of Einstein gravity as the macro-geometry class follows from the application of Lovelock's theorem to the coarse-grained stress-energy tensor. The four geometric constraints—locality, covariance, second-order derivatives, and identity divergence-freeness—that Lovelock's theorem requires are all satisfied by the closure coarse-graining map, forcing the Einstein field equations as the unique consistent gravitational law in four dimensions. All six classical tests of general relativity are recovered.

The framework further provides natural mechanisms for resolving the Hubble tension (via epoch-dependent drift of the loop-gas filling fraction generating dynamical dark energy) and the S_8 tension (via residual pressure of the near-zero-temperature loop gas suppressing late-time structure growth). Both mechanisms arise from the same dual action, offering a unified origin for phenomena that in ΛCDM require independent explanations.

Three critical bottlenecks remain before the theory reaches theorem-grade status: the formal classification theorem for loop action invariants, the derivation of the filling-fraction stability condition from substrate dynamics, and the first-principles computation of the Hagedorn density of states from the worldsheet conformal field theory of S[Ψ]. Each bottleneck has a well-defined mathematical program, and the resolution of all three would constitute a complete, first-principles reconstruction of classical general relativity and its cosmological content from closure-ontological primitives.

The closure program represents a contribution to the ongoing project of understanding why the laws of gravity take the specific form they do. The answer it offers is ontological: spacetime geometry is not a background stage but the cumulative bookkeeping record of the universe's most elementary process—the persistent registration of difference, contact, and conservation at every boundary in the substrate. General relativity, in this view, is not a fortunate discovery but an inevitable result.

APPENDIX A: ZETA-FUNCTION REGULARIZATION OF THE NAMBU-GOTO GROUND STATE

The zero-point energy of a closed Nambu-Goto string in D spacetime dimensions can be computed via zeta-function regularization of the worldsheet harmonic oscillator spectrum. The closed string has mode expansion:

$$X^\mu(\tau, \sigma) = x^\mu + (p^\mu/\pi\sigma_T)\, \tau + (i/\sqrt{2\pi\sigma_T}) \sum_{n\neq 0} (1/n)(\alpha^\mu_n e^{-2in(\tau-\sigma)} + \tilde{\alpha}^\mu_n e^{-2in(\tau+\sigma)})$$

The worldsheet Hamiltonian receives contributions from both left- and right-moving modes. For each physical (transverse) direction, the zero-point contribution is:

$$E_{ZP}^{(1D)} = (1/2) \sum_{n=1}^{\infty} n = (1/2)\, \zeta(-1) = -1/24$$

where $\zeta(-1) = -1/12$ is the Riemann zeta function at $s = -1$. For $D - 2$ transverse directions, the total zero-point energy is $E_{ZP} = -(D-2)/24$ (per sector). In $D = 4$, $E_{ZP} = -1/12$. However, this energy scales with the loop size as $E_{ZP} \propto 1/L \propto V^{-1/3}$, giving $p = -(dE_{ZP}/dV) = -(1/3)\, E_{ZP}/V$ and therefore $w = p/(E_{ZP}/V) = +1/3$. This confirms that the Nambu-Goto ground state behaves as radiation, motivating the need for the S_{bulk} term.

APPENDIX B: EXPLICIT LOVELOCK CONSTRAINT VERIFICATION

We verify that the four conditions of Lovelock's theorem are satisfied by the closure coarse-graining map.

Condition 1 — Locality: The coarse-graining kernel $W(x, x')$ has compact support in spacetime; it falls off exponentially on scales larger than the mesoscale volume $\Omega^{1/4}$. The resulting $T_{\mu\nu}(x)$ therefore depends only on the loop density in a neighborhood of x, satisfying the locality requirement.

Condition 2 — Diffeomorphism Covariance: The action $S[\Psi]$ is constructed from covariant objects: the induced metric $h_{\alpha\beta}$ (a pull-back of $g_{\mu\nu}$), the volume form $\sqrt{-g}$, and the diffeomorphism-invariant step function θ_{loop}. The resulting $T_{\mu\nu}$ transforms as a covariant rank-2 tensor under general coordinate transformations.

Condition 3 — Second-Order Derivatives: $T_{\mu\nu}$ from S_{NG} involves at most first derivatives of $g_{\mu\nu}$ (through $h_{\alpha\beta}$), and $T_{\mu\nu}$ from S_{bulk} involves $g_{\mu\nu}$ algebraically. Neither sector produces terms with second or higher derivatives of the metric in the low-energy limit. (Higher-derivative terms arise only from the extrinsic-curvature corrections, which are suppressed by additional powers of R_0/l_{Pl}.)

Condition 4 — Identity Divergence-Free: The closure coarse-graining kernel is constructed to be identically divergence-free (Section 2.4, condition 2). Therefore $\nabla^\mu T_{\mu\nu} = 0$ for any background metric $g_{\mu\nu}$, not merely on solutions of the field equations. This is the most stringent condition and is the direct analog of the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$.

All four conditions being satisfied, Lovelock's theorem applies and uniquely identifies the geometric tensor $G_{\mu\nu} + \Lambda g_{\mu\nu}$ as the only consistent coupling partner for the closure stress-energy tensor in (3+1) dimensions.

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